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The standard model on non-commutative space-time

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Abstract. We consider the standard model on a non-commutative space and expand the action in the non-commutativity parameter $\theta^{\mu\nu}$. No new particles are introduced; the structure group is $SU(3)\times SU(2)\times U(1)$. We derive the leading order action. At zeroth order the action coincides with the ordinary standard model. At leading order in $\theta^{\mu\nu}$ we find new vertices which are absent in the standard model on commutative space-time. The most striking features are couplings between quarks, gluons and electroweak bosons and many new vertices in the charged and neutral currents. We find that parity is violated in non-commutative QCD. The Higgs mechanism can be applied. QED is not deformed in the minimal version of the NCSM to the order considered.

1 Introduction

A method for implementing non-Abelian $SU(N)$ Yang–Mills theories on non-commutative space-time has recently been proposed [1–4]. Previously only $U(N)$ gauge theories were under control, and it was thus only possible to consider extensions of the standard model. Recently there has been a lot of activity on model building. The aim of this paper is to apply the method proposed in [1–4] to the full standard model of particle physics [5]. We present a minimal non-commutative standard model with structure group $SU(3)_C\times SU(2)_\mathrm{L}\times U(1)_Y$ and with the same fields and the same number of coupling parameters as in the standard model.

On a non-commutative space-time, space-time coordinates do not commute. A particularly simple example is that of a canonical structure

$$[x^\mu \stackrel{\star}{,} x^\nu] \equiv x^\mu \star x^\nu - x^\nu \star x^\mu = \mathrm{i}\theta^{\mu\nu}, \qquad (1)$$

with a constant antisymmetric matrix $\theta^{\mu\nu}$. We may think of $\theta^{\mu\nu}$ as a background field relative to which directions in space-time are distinguished. We use the symbol $\star$ in (1) to denote the product of the non-commutative structure. We shall focus on the case where this structure is given by a star product (see below), because then the discussion of a classical (commutative) limit is particularly transparent.

Obviously, the physics on such a space-time is very different from that on commutative space-time. For example, Lorentz symmetry is explicitly violated. There are several motivations to impose such a relation, which is reminiscent of the non-commutative relation imposed in quantum mechanics between coordinates and momenta. One may speculate that space-time becomes non-commutative at very short distances when quantum gravitation becomes relevant. We would like to point out, however, that the non-commutativity scale could be much lower. An example of a system where space-time coordinates do not commute is that of a particle in a strong magnetic field; see e.g. [6]. Applying similar concepts to particle physics thus does not seem too unnatural. Another motivation comes from string theory where non-commutative gauge theory appears as a certain limit in the presence of a background field B [7]. Moreover, it is very satisfactory to understand how symmetries can arise in a low energy theory like the standard model from a larger theory which is less symmetric. Indeed the non-commutative version of the standard model is Lorentz violating, but the Seiberg–Witten map allows one to understand why Lorentz symmetry is an almost exact symmetry of Nature: the zeroth order of the theory is the Lorentz invariant standard model.

If one is willing to apply the mechanism proposed in [1–4, 8] to the standard model, two problems have to be addressed. First, it has been claimed that in non-commutative quantum electrodynamics charges are quantized and can only take the values ± 1 and zero [9]. This would indeed be a problem in view of the range of hypercharges in the standard model; see Table 1. We will argue that this is really a problem concerning the number of degrees of freedom and will show how it can be overcome with the help of the Seiberg–Witten map. In fact the solution to the problem is closely related to the problem of arbitrary structure groups that we already mentioned. Secondly, we have to deal with tensor products of gauge groups. A no-go theorem concerning this issue has been proposed [10], but we will show that this can again be dealt with using the methods proposed in [1–4, 8]. We incorporate all gauge fields into one "master field" thereby insuring gauge invariance of the theory. There remains some ambiguity in the choice of kinetic terms for the gauge potentials and for the Higgs that we shall discuss.

We expand the non-commutative action in terms of the parameter $\theta^{\mu\nu}$. This expansion corresponds to an expansion in the transferred momentum. It gives a low energy effective action valid for small momentum transfer and it can be compared to the low energy effective theory of quantum chromodynamics known as chiral perturbation theory (see e.g. [11] for a review on chiral perturbation theory). At zeroth order in $\theta^{\mu\nu}$ we recover the ordinary standard model.

A priori there is no reason to expect that $\theta^{\mu\nu}$ is constant. There is no fundamental theoretical obstacle to formulate the theory also for non-constant $\theta^{\mu\nu}(x)$, but we shall concentrate on the constant case in the following for simplicity of presentation. Up to terms involving derivatives of $\theta^{\mu\nu}(x)$, our leading order results are also valid for a slowly varying space-time dependent $\theta^{\mu\nu}(x)$. Furthermore, we should note that there are unresolved problems with unitarity in field theories with non-trivial temporal non-commutativity, so one has to treat that case with care.

One of the main motivations for applying the techniques of [1–4, 8] to the full standard model is to verify that the theory is still consistent with the Higgs mechanism [12]. The Higgs mechanism has previously been discussed in the context of non-commutative Abelian gauge theory [13]. We find that as expected the Higgs mechanism can be applied in the non-commutative version of the standard model. The photon remains massless to all orders in the deformation parameter. In a non-commutative setting the photon can couple to neutral particles via a $\star$-commutator. However, in the minimal version of the NCSM that we present in the main part of this paper we, e.g., do not find a vertex with two Higgs bosons and one electromagnetic photon to any order in the deformation parameter.

2 Gauge theory on non-commutative space-time

The subject has a long history. The idea that coordinates may not commute can be traced back to Heisenberg. For an early reference on field theory on a non-commutative space; see [14]. The mathematical development of non-commutative geometry also has a long history [15]. An interpretation of the electroweak sector in terms of non-commutative geometry has been proposed by Connes and Lott [16]. This is not the topic of the present work. Neither do we consider deformations of the structure group and corresponding quantum gauge theories; see, e.g., [17]. Our aim is to adapt the standard model to the situation where space-time is non-commutative. We are in particular interested in field theory aspects of the type of non-commutative gauge theory that has been a recent focus of interest in string and M(atrix) theory [7]. For a review and more references see, e.g., [18].

We would like to start by briefly reviewing an intuitive approach to the construction of gauge theories over a given non-commutative structure [1]. As such we consider a non-commutative associative algebra $\mathcal{A}$ whose elements we shall call "functions on non-commutative space-time" in the spirit of the Gel'fand–Naimark theorem. For the purposes of this article we shall also require that there is an invariant integral (trace), a well-defined classical limit and that a perturbative treatment of the non-commutativity is possible. This is the case for the canonical structure (1), which can be extended to the Moyal–Weyl star product defined by a formal power series expansion of

$$(f \star g)(x) = \exp\left(\frac{\mathrm{i}}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\right) f(x)g(y)\bigg|_{y\to x}, \quad (2)$$

together with the ordinary integral $\int \mathrm{d}^n x f(x)$. The latter has the property

$$\int \mathrm{d}^n x (f\star g)(x) = \int \mathrm{d}^n x (g\star f)(x) = \int \mathrm{d}^n x f(x)g(x), \quad (3)$$

as can be seen by partial integration. Here $f(x)$, $g(x)$ are ordinary functions on $\mathbb{R}^n$ and the expansion in the star product can be seen intuitively as an expansion of the product in its non-commutativity. One should note that $\mathbb{R}^n$ is only an auxiliary space needed to define the star product. It should not be confused with the "non-commutative space-time" itself, which in contrast to $\mathbb{R}^n$ does not have "points". In the classical limit $\theta^{\mu\nu} \to 0$ we recover ordinary commutative space-time.

2.1 Gauge fields on non-commutative space-time

The construction of a gauge theory on a given non-commutative space can be based on a few basic ideas: the concept of covariant coordinates/functions, the requirement of locality, and gauge equivalence and consistency conditions.

Non-commutative gauge transformations

Let us consider an infinitesimal non-commutative local gauge transformation $\hat\delta$ of a fundamental matter field that carries a representation ρ_Ψ

$$\hat\delta\widehat{\Psi} = \mathrm{i}\rho_\Psi(\widehat{\Lambda}) \star \widehat{\Psi}. \quad (4)$$

In the Abelian case the representation is fixed by the hypercharge. In the non-Abelian case $\widehat{\Psi}$ is a vector, $\rho_\Psi(\widehat{\Lambda})$ a matrix whose entries are functions on non-commutative space-time and $\star$ includes matrix multiplication, i.e., $[\rho_\Psi(\widehat{\Lambda}) \star \widehat{\Psi}]_a \equiv \sum_b [\rho_\Psi(\widehat{\Lambda})]_{ab} \star \widehat{\Psi}_b$.

The product of a field and a coordinate, $\widehat{\Psi} \star x^\mu$, transforms just like $\widehat{\Psi}$, but the opposite product, $x^\mu \star \widehat{\Psi}$, is not a covariant object because the gauge parameter does not commute with x^μ. In complete analogy to the covariant derivatives of ordinary gauge theory we thus need to introduce covariant coordinates $X^\mu = x^\mu + \theta^{\mu\nu}\widehat{A}_\nu$, where

$\widehat{A}_\nu$ is a non-commutative analog of the gauge potential with the following transformation property[1]:

$$\hat{\delta}\widehat{A}_\mu = \partial_\mu \widehat{\Lambda} + \mathrm{i}[\widehat{\Lambda} \stackrel{\star}{,} \widehat{A}_\mu]. \tag{5}$$

From the covariant coordinates one can construct further covariant objects including the non-commutative field strength

$$\widehat{F}_{\mu\nu} = \partial_\mu \widehat{A}_\nu - \partial_\nu \widehat{A}_\mu - \mathrm{i}[\widehat{A}_\mu \stackrel{\star}{,} \widehat{A}_\nu], \quad \hat{\delta}\widehat{F}_{\mu\nu} = \mathrm{i}[\widehat{\Lambda} \stackrel{\star}{,} \widehat{F}_{\mu\nu}], \tag{6}$$

related to the commutator of covariant coordinates, and the covariant derivative

$$\widehat{D}_\mu \widehat{\Psi} = \partial_\mu \widehat{\Psi} - \mathrm{i}\rho_\Psi(\widehat{A}_\mu) \star \widehat{\Psi}, \tag{7}$$

related to the covariant expression $\rho_\Psi(X^\mu) \star \widehat{\Psi} - \widehat{\Psi} \star x^\mu$.

In the following we shall often omit the symbol ρ_Ψ, when its presence is obvious.

Locality, classical limit and Seiberg–Witten maps

A star product of ordinary functions f, g can be seen as a tower built upon its classical limit, which is determined by a Poisson tensor $\theta^{\mu\nu}(x)$,

$$f \star g = f \cdot g + \frac{\mathrm{i}}{2}\theta^{\mu\nu}(x)\partial_\mu f \cdot \partial_\nu g + \mathcal{O}(\theta^2), \tag{8}$$

with higher order terms chosen in such a way as to yield an associative product. The star product is a local function of f, g, meaning that it is a formal series that at each order in θ depends on f, g and a *finite* number of derivatives of f and g.

The non-commutative fields $\widehat{A}$, $\widehat{\Psi}$ and non-commutative gauge parameter $\widehat{\Lambda}$ can be expressed in a similar fashion as towers built upon the corresponding ordinary fields A, Ψ and ordinary gauge parameter Λ. There are so-called Seiberg–Witten maps [7] that express the non-commutative fields and parameters as local functions of the ordinary fields and parameters,

$$\widehat{A}_\xi[A] = A_\xi + \frac{1}{4}\theta^{\mu\nu}\{A_\nu, \partial_\mu A_\xi\} + \frac{1}{4}\theta^{\mu\nu}\{F_{\mu\xi}, A_\nu\} + \mathcal{O}(\theta^2), \tag{9}$$

$$\widehat{\Psi}[\Psi, A] = \Psi + \frac{1}{2}\theta^{\mu\nu}\rho_\Psi(A_\nu)\partial_\mu \Psi + \frac{\mathrm{i}}{8}\theta^{\mu\nu}[\rho_\Psi(A_\mu), \rho_\Psi(A_\nu)]\Psi + \mathcal{O}(\theta^2), \tag{10}$$

$$\widehat{\Lambda}[\Lambda, A] = \Lambda + \frac{1}{4}\theta^{\mu\nu}\{A_\nu, \partial_\mu \Lambda\} + \mathcal{O}(\theta^2), \tag{11}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}[A_\mu, A_\nu]$ is the ordinary field strength. We shall henceforth omit the explicit dependence of the non-commutative fields and parameters on their ordinary counterparts with the understanding that the hat $\widehat{\ }$ denotes non-commutative quantities that can be expanded as local functions of their classical counterparts via Seiberg–Witten maps.

Gauge equivalence and consistency condition

The Seiberg–Witten maps have the remarkable property that ordinary gauge transformations $\delta A_\mu = \partial_\mu \Lambda + \mathrm{i}[\Lambda, A_\mu]$ and $\delta\Psi = \mathrm{i}\Lambda \cdot \Psi$ induce non-commutative gauge transformations (4), (5) of the fields $\widehat{A}$, $\widehat{\Psi}$ with the gauge parameter $\widehat{\Lambda}$ as given above:

$$\delta\widehat{A}_\mu = \hat{\delta}\widehat{A}_\mu, \quad \delta\widehat{\Psi} = \hat{\delta}\widehat{\Psi}. \tag{12}$$

For consistency we have to require that any pair of non-commutative gauge parameters $\widehat{\Lambda}$, $\widehat{\Sigma}$ satisfy

$$[\widehat{\Lambda} \stackrel{\star}{,} \widehat{\Sigma}] + \mathrm{i}\delta_\Lambda \widehat{\Sigma} - \mathrm{i}\delta_\Sigma \widehat{\Lambda} = \widehat{[\Lambda, \Sigma]}. \tag{13}$$

Since this consistency condition involves solely the gauge parameters it is convenient to base the construction of the Seiberg–Witten map (11) on it. In a second step the remaining Seiberg–Witten maps (9) and (10) can be computed from the gauge equivalence condition (12). The gauge equivalence and consistency conditions do not uniquely determine Seiberg–Witten maps. To the order considered here we have the freedom of classical field redefinitions and non-commutative gauge transformations. We have used the latter freedom to choose Seiberg–Witten maps with hermitian $\widehat{\Lambda}$ and $\widehat{A}_\mu$.

The freedom in the Seiberg–Witten map is essential for the renormalization of non-commutative gauge theory [8]. The constants that parametrize the freedom in the Seiberg–Witten map become free running coupling constant which are determined by the unknown fundamental theory which is responsible for the non-commutative nature of space-time. The field redefinition freedom is also important in the context of tensor products of gauge groups.

2.2 Non-Abelian gauge groups

The commutator

$$[\widehat{\Lambda} \stackrel{\star}{,} \widehat{\Lambda}'] = \frac{1}{2}\{\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)\}[T^a, T^b] + \frac{1}{2}[\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)]\{T^a, T^b\} \tag{14}$$

of two Lie algebra-valued non-commutative gauge parameters $\widehat{\Lambda} = \Lambda_a(x)T^a$ and $\widehat{\Lambda}' = \Lambda'_a(x)T^a$ does not close in the Lie algebra. It is in general enveloping algebra-valued (it contains products of generators), because the coefficient $[\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)]$ of the anti-commutator of generators $\{T^a, T^b\}$ is in general non-zero in the non-commutative case [1,2]. An important exception is $U(N)$ in the fundamental representation. If we try, however, to construct

[1] Here and in the following we use $\theta^{\mu\nu}$ to lower indices, yielding expressions that are more convenient to work with. We should note that this is in general only possible in the case of constant $\theta^{\mu\nu}$

non-commutative $SU(N)$ with Lie algebra-valued gauge parameters, we immediately face the problem that a tracelessness condition is incompatible with (14). We thus have to consider enveloping algebra-valued non-commutative gauge parameters

$$\widehat{\Lambda} = \Lambda^0_a(x)T^a + \Lambda^1_{ab}(x) : T^aT^b : + \Lambda^2_{abc}(x) : T^aT^bT^c : + \ldots \quad (15)$$

and fields. (The symbol : : denotes some appropriate ordering of the Lie algebra generators.) A priori we now face the problem that we have an infinite number of parameters $\Lambda^0_a(x)$, $\Lambda^1_{ab}(x)$, $\Lambda^2_{abc}(x)$, $\ldots$, but these are not independent. They can in fact all be expressed in terms of the right number of classical parameters and fields via the Seiberg–Witten maps. Similar observations and conclusions hold for the non-commutative non-Abelian gauge fields.

2.3 Charge in non-commutative QED

In non-commutative QED one faces the problem that the theory can apparently accommodate only charges $\pm q$ or zero for one fixed q [9]. We shall briefly review the problem below and will argue that there is no such restriction in the θ-expanded approach based on Seiberg–Witten maps. The problem (and its solution) is in fact related to the problem with arbitrary gauge groups that we discussed above: The commutation of Lie algebra-valued non-commutative gauge parameters closes only in the fundamental representation of $U(1)$.

The only couplings of the non-commutative gauge boson $\widehat{A}_\mu$ to a matter field $\widehat{\Psi}$ compatible with the non-commutative gauge transformation (5) in addition to (7) are

$$\widehat{D}^-_\mu\widehat{\Psi}^- = \partial_\mu\widehat{\Psi}^- + \mathrm{i}\widehat{\Psi}^- \star \widehat{A}_\mu, \quad \widehat{D}^0_\mu\widehat{\Psi}^0 = \partial_\mu\widehat{\Psi}^0,$$
$$\widehat{D}^{0'}_\mu\widehat{\Psi}^{0'} = \partial_\mu\widehat{\Psi}^{0'} - \mathrm{i}[\widehat{A}_\mu \stackrel{\star}{,} \widehat{\Psi}^{0'}], \quad (16)$$

with $\hat{\delta}\widehat{\Psi}^- = -\mathrm{i}\widehat{\Psi}^- \star \widehat{\Lambda}$, $\hat{\delta}\widehat{\Psi}^0 = 0$, and $\hat{\delta}\widehat{\Psi}^{0'} = \mathrm{i}[\widehat{\Lambda} \stackrel{\star}{,} \widehat{\Psi}^{0'}]$, respectively. (The latter possibility is interesting since it shows how a neutral particle can couple to the (hyper) photon in a non-commutative setting [19].) At first sight, it thus appears that only $U(1)$ charges $+1$, -1, 0 are possible.

We should of course consider physical fields $\hat{a}^{(n)}_\mu(x)$. Let Q be the generator of $U(1)$ (charge operator), e a coupling constant and $\psi^{(n)}$ a field for a particle of charge $q^{(n)}$. Then $A_\mu = eQa_\mu(x)$ and $\hat{A}_\mu \star \hat{\psi}^{(n)} = eq^{(n)}\hat{a}^{(n)}_\mu(x) \star \hat{\psi}^{(n)}$, since the Seiberg–Witten map $\hat{A}_\mu$ depends explicitly on Q. In ordinary QED there is only one photon, i.e., there is no need for a label (n) on a_μ. Here, however, we have a separate $\hat{a}^{(n)}_\mu$ for every charge $q^{(n)}$ in the theory. The field strength

$$\widehat{f}^{(n)}_{\mu\nu} = \partial_\mu\widehat{a}^{(n)}_\nu - \partial_\nu\widehat{a}^{(n)}_\mu + \mathrm{i}eq^{(n)}[\widehat{a}^{(n)}_\mu \stackrel{\star}{,} \widehat{a}^{(n)}_\nu] \quad (17)$$

and covariant derivative

$$\widehat{D}_\mu\widehat{\psi}^{(n)} = \partial_\mu\widehat{\psi}^{(n)} - \mathrm{i}eq^{(n)}\widehat{a}^{(n)}_\mu\widehat{\psi}^{(n)} \quad (18)$$

transform covariantly under

$$\hat{\delta}\widehat{a}^{(n)}_\mu = \partial_\mu\widehat{\lambda}^{(n)} + \mathrm{i}eq^{(n)}[\widehat{\lambda}^{(n)} \stackrel{\star}{,} \widehat{a}^{(n)}_\mu],$$
$$\hat{\delta}\widehat{\psi}^{(n)} = \mathrm{i}eq^{(n)}\widehat{\lambda}^{(n)} \star \widehat{\psi}^{(n)}. \quad (19)$$

We see that the $\hat{a}^{(n)}_\mu$ cannot be equal to each other because of the non-zero $\star$-commutator in the transformation of $\hat{a}^{(n)}_\mu$. It is not possible to absorb $q^{(n)}$ in a redefinition of $\hat{a}^{(n)}_\mu$.

We can have any charge now, but it appears that we have too many degrees of freedom. This is not really the case, however, since all $\hat{a}^{(n)}_\mu$ are local functions of the correct number of classical gauge fields a_μ via the Seiberg–Witten map (9) that, when written in terms of the physical fields, depends on $q^{(n)}$:

$$\hat{a}^{(n)}_\xi = a_\xi + \frac{eq^{(n)}}{4}\theta^{\mu\nu}\{a_\nu, \partial_\mu a_\xi\} + \frac{eq^{(n)}}{4}\theta^{\mu\nu}\{f_{\mu\xi}, a_\nu\} + \mathcal{O}(\theta^2). \quad (20)$$

In the action for the non-commutative gauge fields we now face a choice: From the non-commutative point of view it appears to be natural to provide kinetic terms for all $\hat{a}^{(n)}_\mu$, even though these fields are not really independent. This leads to a trace over the particles in the model and will be discussed in Appendix C. In the main part of this paper we will instead make a simpler choice for the trace that leads to minimal deviations from the ordinary standard model. That choice is more natural from the point of view that the independent degrees of freedom are given by the a_μ. Gauge invariance alone is not enough to favor one of the possible choices.

2.4 Non-commutative Yukawa couplings and Higgs

We can generalize (10) to the case of a field Φ that transforms on the left and on the right under two arbitrary gauge groups with corresponding gauge potentials A_μ, A'_μ. We have $\widehat{\Phi} \equiv \widehat{\Phi}[\Phi, A, A']$, given by the following hybrid Seiberg–Witten map:

$$\widehat{\Phi}[\Phi, A, A'] = \Phi + \frac{1}{2}\theta^{\mu\nu}A_\nu\left(\partial_\mu\Phi - \frac{\mathrm{i}}{2}(A_\mu\Phi - \Phi A'_\mu)\right) + \frac{1}{2}\theta^{\mu\nu}\left(\partial_\mu\Phi - \frac{\mathrm{i}}{2}(A_\mu\Phi - \Phi A'_\mu)\right)A'_\nu + \mathcal{O}(\theta^2). \quad (21)$$

It transforms covariantly,

$$\delta\widehat{\Phi} = \mathrm{i}\widehat{\Lambda} \star \widehat{\Phi} - \mathrm{i}\widehat{\Phi} \star \widehat{\Lambda}', \quad (22)$$

under $\delta\Phi = \mathrm{i}\Lambda\Phi - \mathrm{i}\Phi\Lambda'$, $\delta A_\nu = \partial_\nu\Lambda + \mathrm{i}[\Lambda, A_\nu]$, $\delta A'_\nu = \partial_\nu\Lambda' + \mathrm{i}[\Lambda', A'_\nu]$. The covariant derivative for $\widehat{\Phi}$ is

$$\widehat{D}_\mu\widehat{\Phi} = \partial_\mu\widehat{\Phi} - \mathrm{i}\widehat{A}_\mu \star \widehat{\Phi} + \mathrm{i}\widehat{\Phi} \star \widehat{A'}_\mu. \quad (23)$$

We need the hybrid Seiberg–Witten map to construct gauge covariant Yukawa couplings. The classical ("commutative") Higgs Φ has $U(1)$ charge $Y = 1/2$ and transforms under $SU(2)$ in the fundamental representation. It has no color charge. Φ obviously commutes with the classical $U(1)$ and $SU(3)$ gauge parameters. In the non-commutative case this is not the case because both Φ and the parameters are functions on space-time and thus do not commute. It is still true that the non-commutative $\widehat{\Phi}$ has overall $U(1)$ charge $Y = 1/2$ and no overall color charge, but the precise representations on the left (affects A_μ) and on the right (affects A'_μ) are inherited from the fermions on the left and the right of the Higgs in the Yukawa couplings.

3 The non-commutative standard model

The structure group of the standard model is $G_{\mathrm{SM}} = SU(3)_C \times SU(2)_{\mathrm{L}} \times U(1)_Y$. There are several ways to deal with this tensor product in the non-commutative case that correspond to a freedom in the choice of Seiberg–Witten map. The simplest, symmetric and most natural approach is to take the classical tensor product and consider the whole gauge potential V_μ of G_{SM} as defined by

$$V_\nu = g'\mathcal{A}_\nu(x)Y + g\sum_{a=1}^{3} B_{\nu a}(x)T^a_{\mathrm{L}} + g_S\sum_{b=1}^{8} G_{\nu b}(x)T^b_S \quad (24)$$

and the commutative gauge parameter Λ by

$$\Lambda = g'\alpha(x)Y + g\sum_{a=1}^{3}\alpha^L_a(x)T^a_{\mathrm{L}} + g_S\sum_{b=1}^{8}\alpha^S_b(x)T^b_S, \quad (25)$$

where Y, T^a_{L}, T^b_S are the generators of $u(1)_Y$, $su(2)_{\mathrm{L}}$ and $su(3)_C$ respectively. The non-commutative gauge parameter $\widehat{\Lambda}$ is then given via the Seiberg–Witten map by

$$\widehat{\Lambda} = \Lambda + \frac{1}{4}\theta^{\mu\nu}\{V_\nu, \partial_\mu\Lambda\} + \mathcal{O}(\theta^2). \quad (26)$$

Note that this is not equal to a naive sum of non-commutative gauge parameters corresponding to the three factors in G_{SM}. This is due to the nonlinearity of the Seiberg–Witten maps and ultimately is a consequence of the nonlinear consistency condition (13). The non-commutative fermion fields $\widehat{\Psi}^{(n)}$ corresponding to particles labelled by (n) is

$$\widehat{\Psi}^{(n)} = \Psi^{(n)} + \frac{1}{2}\theta^{\mu\nu}\rho_{(n)}(V_\nu)\partial_\mu\Psi^{(n)} + \frac{\mathrm{i}}{8}\theta^{\mu\nu}[\rho_{(n)}(V_\mu), \rho_{(n)}(V_\nu)]\Psi^{(n)} + \mathcal{O}(\theta^2). \quad (27)$$

The Seiberg–Witten map for the non-commutative vector potential $\widehat{V}_\mu$ yields

$$\widehat{V}_\xi = V_\xi + \frac{1}{4}\theta^{\mu\nu}\{V_\nu, \partial_\mu V_\xi\} + \frac{1}{4}\theta^{\mu\nu}\{F_{\mu\xi}, V_\nu\} + \mathcal{O}(\theta^2), \quad (28)$$

Table 1. The standard model fields. The electric charge is given by the Gell-Mann-Nishijima relation $Q = (T_3 + Y)$. The fields B^i with $i \in \{+,-,3\}$ denote the three electroweak gauge bosons. The gluons G^i are in the octet representation of $SU(3)_C$

	$SU(3)_C$	$SU(2)_{\mathrm{L}}$	$U(1)_Y$	$U(1)_Q$
e_{R}	**1**	**1**	-1	-1
$L_{\mathrm{L}} = \begin{pmatrix}\nu_{\mathrm{L}}\\ e_{\mathrm{L}}\end{pmatrix}$	**1**	**2**	$-1/2$	$\begin{pmatrix}0\\ -1\end{pmatrix}$
u_{R}	**3**	**1**	$2/3$	$2/3$
d_{R}	**3**	**1**	$-1/3$	$-1/3$
$Q_{\mathrm{L}} = \begin{pmatrix}u_{\mathrm{L}}\\ d_{\mathrm{L}}\end{pmatrix}$	**3**	**2**	$1/6$	$\begin{pmatrix}2/3\\ -1/3\end{pmatrix}$
$\Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$	**1**	**2**	$1/2$	$\begin{pmatrix}1\\ 0\end{pmatrix}$
B^i	**1**	**3**	0	$(\pm1, 0)$
A	**1**	**1**	0	0
G^a	**8**	**1**	0	0

with the ordinary field strength $F^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu - \mathrm{i}[V^\mu, V^\nu]$. The non-commutative field strength is

$$\widehat{F}_{\mu\nu} = \partial_\mu\widehat{V}_\nu - \partial_\nu\widehat{V}_\mu - \mathrm{i}[\widehat{V}_\mu \stackrel{*}{,} \widehat{V}_\nu]. \quad (29)$$

We have the following particle spectrum; see Table 1:

$$\Psi^{(i)}_{\mathrm{L}} = \begin{pmatrix}L^{(i)}_{\mathrm{L}}\\ Q^{(i)}_{\mathrm{L}}\end{pmatrix}, \quad \Psi^{(i)}_{\mathrm{R}} = \begin{pmatrix}e^{(i)}_{\mathrm{R}}\\ u^{(i)}_{\mathrm{R}}\\ d^{(i)}_{\mathrm{R}}\end{pmatrix}, \quad \Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}, \quad (30)$$

where $(i) \in \{1,2,3\}$ is the generation index and ϕ^+ and ϕ^0 are the complex scalar fields of the scalar Higgs doublet. The non-commutative Higgs field $\widehat{\Phi}$ is given by the hybrid Seiberg–Witten map (21),

$$\widehat{\Phi} = \Phi + \frac{1}{2}\theta^{\mu\nu}V_\nu\left(\partial_\mu\Phi - \frac{\mathrm{i}}{2}(V_\mu\Phi - \Phi V'_\mu)\right) + \frac{1}{2}\theta^{\mu\nu}\left(\partial_\mu\Phi - \frac{\mathrm{i}}{2}(V_\mu\Phi - \Phi V'_\mu)\right)V'_\nu + \mathcal{O}(\theta^2)\,. \quad (31)$$

The non-commutative standard model can now be written in a very compact way:

$$\begin{aligned} S_{\mathrm{NCSM}} = &\int \mathrm{d}^4x \sum_{i=1}^{3} \overline{\widehat{\Psi}}^{(i)}_{\mathrm{L}} \star \mathrm{i}\widehat{\not{D}}\widehat{\Psi}^{(i)}_{\mathrm{L}} + \int \mathrm{d}^4x \sum_{i=1}^{3} \overline{\widehat{\Psi}}^{(i)}_{\mathrm{R}} \star \mathrm{i}\widehat{\not{D}}\widehat{\Psi}^{(i)}_{\mathrm{R}} \\ &- \int \mathrm{d}^4x \frac{1}{2g'}\mathbf{tr_1}\widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} - \int \mathrm{d}^4x \frac{1}{2g}\mathbf{tr_2}\widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} \\ &- \int \mathrm{d}^4x \frac{1}{2g_S}\mathbf{tr_3}\widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} \\ &+ \int \mathrm{d}^4x \Big(\rho_0(\widehat{D}_\mu\widehat{\Phi})^\dagger \star \rho_0(\widehat{D}^\mu\widehat{\Phi}) - \mu^2\rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi}) \\ &- \lambda\rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi}) \star \rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi})\Big) \end{aligned}$$

$$+ \int \mathrm{d}^4x \Bigg(- \sum_{i,j=1}^{3} W^{ij} \Big((\bar{\widehat{L}}_{\mathrm{L}}^{(i)} \star \rho_{\mathrm{L}}(\widehat{\Phi})) \star \widehat{e}_{\mathrm{R}}^{(j)}$$
$$+ \bar{\widehat{e}}_{\mathrm{R}}^{(i)} \star (\rho_{\mathrm{L}}(\widehat{\Phi})^\dagger \star \widehat{L}_{\mathrm{L}}^{(j)}) \Big)$$
$$- \sum_{i,j=1}^{3} G_u^{ij} \Big((\bar{\widehat{Q}}_{\mathrm{L}}^{(i)} \star \rho_{\bar{Q}}(\widehat{\bar{\Phi}})) \star \widehat{u}_{\mathrm{R}}^{(j)}$$
$$+ \bar{\widehat{u}}_{\mathrm{R}}^{(i)} \star (\rho_{\bar{Q}}(\widehat{\bar{\Phi}})^\dagger \star \widehat{Q}_{\mathrm{L}}^{(j)}) \Big)$$
$$- \sum_{i,j=1}^{3} G_d^{ij} \Big((\bar{\widehat{Q}}_{\mathrm{L}}^{(i)} \star \rho_{Q}(\widehat{\Phi})) \star \widehat{d}_{\mathrm{R}}^{(j)}$$
$$+ \bar{\widehat{d}}_{\mathrm{R}}^{(i)} \star (\rho_{Q}(\widehat{\Phi})^\dagger \star \widehat{Q}_{\mathrm{L}}^{(j)}) \Big) \Bigg), \quad (32)$$

with $\bar{\Phi} = \mathrm{i}\tau_2 \Phi^*$. The matrices W^{ij}, G_u^{ij} and G_d^{ij} are the Yukawa couplings. The gauge fields in the Seiberg–Witten maps and covariant derivatives of the fermions terms are summarized in Table 2. The representation used in the trace of the kinetic terms for the gauge bosons is not uniquely determined by gauge invariance of the action. We pick the simplest choice of a sum of traces over the $U(1)$, $SU(2)$ and $SU(3)$ sectors, because we are interested in a version of the standard model on non-commutative space-time with minimal modifications[2]. In this spirit we also take a simple choice of representation of Y of the form

$$Y = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (33)$$

in the definition of $\mathbf{tr_1}$. The traces $\mathbf{tr_2}$ and trace $\mathbf{tr_3}$ are the usual $SU(2)$, respectively $SU(3)$ traces. The representations ρ_{L}, ρ_Q, $\rho_{\bar{Q}}$ of the gauge potentials V_μ, V'_μ that appear in the hybrid Seiberg–Witten map of the Higgs are those of the fermions on the left and right of the Higgs in the Yukawa couplings; see (21),

$$\rho_{\mathrm{L}}(\hat{\Phi}[\phi, V_\mu, V'_\nu]) = \hat{\Phi}\left[\phi, -\frac{1}{2}g'\mathcal{A}_\mu + gB_\mu^a T_{\mathrm{L}}^a, g'\mathcal{A}_\nu\right], \quad (34)$$

$$\rho_Q(\hat{\Phi}[\phi, V_\mu, V'_\nu]) = \hat{\Phi}\left[\phi, \frac{1}{6}g'\mathcal{A}_\mu + gB_\mu^a T_{\mathrm{L}}^a + g_S G_\mu^a T_S^a, \frac{1}{3}g'\mathcal{A}_\nu - g_S G_\nu^a T_S^a\right], \quad (35)$$

$$\rho_{\bar{Q}}(\hat{\Phi}[\phi, V_\mu, V'_\nu]) = \hat{\Phi}\left[\phi, \frac{1}{6}g'\mathcal{A}_\mu + gB_\mu^a T_{\mathrm{L}}^a + g_S G_\mu^a T_S^a, -\frac{2}{3}g'\mathcal{A}_\nu - g_S G_\nu^a T_S^a\right]. \quad (36)$$

The representation ρ_0 of these gauge potentials in the kinetic term of the Higgs and in the Higgs potential is the simplest possible one:

$$\rho_0(\hat{\Phi}[\phi, V_\mu, V'_\nu]) = \hat{\Phi}\left[\phi, \frac{1}{2}g'\mathcal{A}_\mu + gB_\mu^a T_{\mathrm{L}}^a, 0\right]. \quad (37)$$

[2] In Appendix C we present a different choice that is perhaps more natural from the non-commutative point of view, with a trace over the particles in the standard model

Table 2. The gauge fields in the Seiberg–Witten maps of the fermions and in the covariant derivatives of the fermions in the non-commutative standard model. (The symbols T_{L}^a and T_S^b are here the Pauli and Gell-Mann matrices respectively)

$\Psi^{(n)}$	$\rho_{(n)}(V_\nu)$
e_{R}	$-g'\mathcal{A}_\nu(x)$
$L_{\mathrm{L}} = \begin{pmatrix} \nu_{\mathrm{L}} \\ e_{\mathrm{L}} \end{pmatrix}$	$-\frac{1}{2}g'\mathcal{A}_\nu(x) + gB_{\nu a}(x)T_{\mathrm{L}}^a$
u_{R}	$\frac{2}{3}g'\mathcal{A}_\nu(x) + g_S G_{\nu b}(x)T_S^b$
d_{R}	$-\frac{1}{3}g'\mathcal{A}_\nu(x) + g_S G_{\nu b}(x)T_S^b$
$Q_{\mathrm{L}} = \begin{pmatrix} u_{\mathrm{L}} \\ d_{\mathrm{L}} \end{pmatrix}$	$\frac{1}{6}g'\mathcal{A}_\nu(x) + gB_{\nu a}(x)T_{\mathrm{L}}^a + g_S G_{\nu b}(x)T_S^b$

There are many possibilities to choose the representations in the kinetic terms of the gauge bosons.

Here, we decide to single out the choice with minimal deviations from the standard model. In Appendix C we discuss this in more detail and present another natural choice. Eventually physical criteria should single out the right choice. These criteria may include, e.g., renormalization, CPT invariance, anomaly freedom, or any kind of symmetry one might want to impose on the action.

4 The non-commutative electroweak sector

In this section we shall apply the Seiberg–Witten map to the electroweak non-commutative standard model. The gauge group of the model is $SU(3)_C \times SU(2)_{\mathrm{L}} \times U(1)_Y$. The particle content is that of the standard model. The matter fields and gauge fields content is summarized in Table 1.

In the following, we shall work in the leading order of the expansion in θ. In our convention, fields with a hat are non-commutative whereas those without a hat are ordinary fields. In particular, we use the following definitions: $\mathcal{A}_\mu$ is the ordinary $U(1)_Y$ field, $B_\mu = B_\mu^i T_{\mathrm{L}}^i$ are the ordinary $SU(2)_{\mathrm{L}}$ fields and $G_\mu = G_\mu^i T_S^i$ are the ordinary $SU(3)_C$ fields. For the lepton field $L_{\mathrm{L}}^{(i)}$ of the ith generation which is in the fundamental representation of $SU(2)_{\mathrm{L}}$ and in the Y representation of $U(1)_Y$, we have the following expansion:

$$\widehat{L}_{\mathrm{L}}^{(i)}[\mathcal{A}, B] = L_{\mathrm{L}}^{(i)} + L_{\mathrm{L}}^{(i)1}[\mathcal{A}, B] + \mathcal{O}(\theta^2), \quad (38)$$

with

$$L_{\mathrm{L}}^{(i)1}[\mathcal{A}, B] = -\frac{1}{2}g'\theta^{\mu\nu}\mathcal{A}_\mu\partial_\nu L_{\mathrm{L}} - \frac{1}{2}g\theta^{\mu\nu}B_\mu\partial_\nu L_{\mathrm{L}} + \frac{\mathrm{i}}{4}\theta^{\mu\nu}\left(g'\mathcal{A}_\mu + gB_\mu\right)\left(g'\mathcal{A}_\nu + gB_\nu\right)L_{\mathrm{L}}. \quad (39)$$

For a right-handed lepton field of the ith generation, one has

$$\widehat{e}_{\mathrm{R}}^{(i)}[\mathcal{A}] = e_{\mathrm{R}}^{(i)} + e_{\mathrm{R}}^{(i)1}[\mathcal{A}] + \mathcal{O}(\theta^2), \quad (40)$$

with

$$e_{\mathrm{R}}^{(i)1}[\mathcal{A}] = -\frac{1}{2}g'\theta^{\mu\nu}\mathcal{A}_\mu\partial_\nu e_{\mathrm{R}}^{(i)}. \quad (41)$$

We have

$$\widehat{Q}_{\rm L}^{(i)}[\mathcal{A},B,G] = Q_{\rm L}^{(i)} + Q_{\rm L}^{(i)1}[\mathcal{A},B,G] + \mathcal{O}(\theta^2) \tag{42}$$

for a left-handed quark doublet $\widehat{Q}_{\rm L}^{(i)}$ of the ith generation, where

$$\begin{aligned} Q_{\rm L}^{(i)1}[\mathcal{A},B,G] = &-\frac{1}{2}g'\theta^{\mu\nu}\mathcal{A}_\mu\partial_\nu Q_{\rm L} \\ &-\frac{1}{2}g\theta^{\mu\nu}B_\mu\partial_\nu Q_{\rm L} - \frac{1}{2}g_S\theta^{\mu\nu}G_\mu\partial_\nu Q_{\rm L} \\ &+\frac{\rm i}{4}\theta^{\mu\nu}\left(g'\mathcal{A}_\mu + gB_\mu + g_S G_\mu\right) \\ &\times\left(g'\mathcal{A}_\nu + gB_\nu + g_S G_\nu\right)Q_{\rm L}. \end{aligned} \tag{43}$$

For a right-handed quark e.g. $\widehat{u}_{\rm R}^{(i)}$, we have

$$\widehat{u}_{\rm R}^{(i)}[\mathcal{A},G] = u_{\rm R}^{(i)} + u_{\rm R}^{(i)1}[\mathcal{A},G] + \mathcal{O}(\theta^2) \tag{44}$$

$$\begin{aligned} u_{\rm R}^{(i)1}[\mathcal{A},G] = &-\frac{1}{2}g'\theta^{\mu\nu}\mathcal{A}_\mu\partial_\nu u_{\rm R} - \frac{1}{2}g_S\theta^{\mu\nu}G_\mu\partial_\nu u_{\rm R} \\ &+\frac{\rm i}{4}\theta^{\mu\nu}\left(g'\mathcal{A}_\mu + g_S G_\mu\right)\left(g'\mathcal{A}_\nu + g_S G_\nu\right)u_{\rm R}. \end{aligned}$$

The same expansion is obtained for a right-handed down-type quark $d_{\rm R}^{(i)}$.

The field strength $\widehat{F}_{\mu\nu} = \partial_\mu\widehat{V}_\nu - \partial_\nu\widehat{V}_\mu - {\rm i}[\widehat{V}_\mu \stackrel{*}{,} \widehat{V}_\nu]$ has the following expansion:

$$\widehat{F}_{\mu\nu} = F_{\mu\nu} + F^1_{\mu\nu} + \mathcal{O}(\theta^2), \tag{45}$$

with

$$F_{\mu\nu} = g' f_{\mu\nu} + gF^L_{\mu\nu} + g_S F^S_{\mu\nu}, \tag{46}$$

where $f_{\mu\nu}$ is the field strength corresponding to the group $U(1)_Y$, $F^L_{\mu\nu}$ that to $SU(2)_{\rm L}$ and $F^S_{\mu\nu}$ that to $SU(3)_C$. The coupling constants of the gauge groups $U(1)_Y$, $SU(2)_{\rm L}$ and $SU(3)_C$ are respectively denoted by g', g and g_S. The leading order correction in θ is given by

$$F^1_{\mu\nu} = \frac{1}{2}\theta^{\alpha\beta}\{F_{\mu\alpha}, F_{\nu\beta}\} - \frac{1}{4}\theta^{\alpha\beta}\{V_\alpha, (\partial_\beta + D_\beta)F_{\mu\nu}\}, \tag{47}$$

with

$$D_\beta F_{\mu\nu} = \partial_\beta F_{\mu\nu} - {\rm i}[V_\beta, F_{\mu\nu}]. \tag{48}$$

The leading order expansion for the mathematical field V is given by

$$\widehat{V}_\mu = V_\mu + {\rm i}\Gamma_\mu + \mathcal{O}(\theta^2), \tag{49}$$

with

$$\begin{aligned} \Gamma_\mu = {\rm i}\frac{1}{4}\theta^{\alpha\beta}\Big\{&g'\mathcal{A}_\alpha + gB_\alpha + g_S G_\alpha, g'\partial_\beta\mathcal{A}_\mu + g\partial_\beta B_\mu \\ &+ g_S\partial_\beta G_\mu + g' f_{\beta\mu} + gF^L_{\beta\mu} + g_S F^S_{\beta\mu}\Big\}. \end{aligned} \tag{50}$$

The action of the non-commutative electroweak standard model reads

$$\begin{aligned} S_{\rm NCSM} = &S_{\rm Matter,leptons} + S_{\rm Matter,quarks} \\ &+ S_{\rm Gauge} + S_{\rm Higgs} + S_{\rm Yukawa}. \end{aligned} \tag{51}$$

We shall first consider the fermions (leptons and quarks). The fermionic matter part is

$$\begin{aligned} &S_{\rm Matter,fermions} \\ &= \int {\rm d}^4x \left(\sum_f \overline{\widehat{\Psi}}_{f{\rm L}} \star {\rm i}\widehat{D\!\!\!/}\widehat{\Psi}_{f{\rm L}} + \sum_f \overline{\widehat{\Psi}}_{f{\rm R}} \star {\rm i}\widehat{D\!\!\!/}\widehat{\Psi}_{f{\rm R}}\right), \end{aligned} \tag{52}$$

where $\widehat{\Psi}_{\rm L}^{(f)}$ denotes the left-handed $SU(2)$ doublets $\widehat{\Psi}_{\rm R}^{(f)}$ the right-handed $SU(2)$ singlets and the index f runs over the three flavors. We thus have:

$$\Psi_{\rm L}^{(1)} = \begin{pmatrix} \begin{pmatrix} \nu_{\rm L} \\ e_{\rm L} \end{pmatrix} \\ \begin{pmatrix} u^r_{\rm L} \\ d^r_{\rm L} \end{pmatrix} \\ \begin{pmatrix} u^y_{\rm L} \\ d^y_{\rm L} \end{pmatrix} \\ \begin{pmatrix} u^b_{\rm L} \\ d^b_{\rm L} \end{pmatrix} \end{pmatrix}, \quad \Psi_{\rm R}^{(1)} = \begin{pmatrix} e_{\rm R} \\ u^r_{\rm R} \\ d^r_{\rm R} \\ u^y_{\rm R} \\ d^y_{\rm R} \\ u^b_{\rm R} \\ d^b_{\rm R} \end{pmatrix} \tag{53}$$

for the first generation.

We thus have

$$\begin{aligned} S_{\rm Matter,fermions} = &\int {\rm d}^4x \\ &\times\left(\sum_i \left(\bar{L}_{\rm L}^{(i)} + \bar{L}_{\rm L}^{(i)1}\right)\star{\rm i}\left(D\!\!\!/^{\rm SM} + \Gamma\!\!\!/\right)\star\left(L_{\rm L}^{(i)} + L_{\rm L}^{(i)1}\right)\right. \\ &\left.+\sum_i \left(\bar{e}_{\rm R}^{(i)} + \bar{e}_{\rm R}^{(i)1}\right){\rm i}\star\left(D\!\!\!/^{\rm SM} + \Gamma\!\!\!/\right)\star\left(e_{\rm R}^{(i)} + e_{\rm R}^{(i)1}\right)\right) \\ &+\mathcal{O}(\theta^2) \\ &= \int {\rm d}^4x \sum_i \bar{L}_{\rm L}^{(i)}{\rm i}D\!\!\!/^{\rm SM}L_{\rm L}^{(i)} \\ &-\frac{1}{4}\theta^{\mu\nu}\int {\rm d}^4x \sum_i \bar{L}_{\rm L}^{(i)}(g' f_{\mu\nu} + gF^L_{\mu\nu}){\rm i}D\!\!\!/^{\rm SM}L_{\rm L}^{(i)} \\ &-\frac{1}{2}\theta^{\mu\nu}\int {\rm d}^4x \sum_i \bar{L}_{\rm L}^{(i)}\gamma^\alpha(g' f_{\alpha\mu} + gF^L_{\alpha\mu}){\rm i}D^{\rm SM}_\nu L_{\rm L}^{(i)} \\ &+\int {\rm d}^4x \sum_i \bar{e}_{\rm R}^{(i)}{\rm i}D\!\!\!/^{\rm SM}e_{\rm R}^{(i)} \\ &-\frac{1}{4}\theta^{\mu\nu}\int {\rm d}^4x \sum_i \bar{e}_{\rm R}^{(i)}g' f_{\mu\nu}{\rm i}D\!\!\!/^{\rm SM}e_{\rm R}^{(i)} \\ &-\frac{1}{2}\theta^{\mu\nu}\int {\rm d}^4x \sum_i \bar{e}_{\rm R}^{(i)}\gamma^\alpha g' f_{\alpha\mu}{\rm i}D^{\rm SM}_\nu e_{\rm R}^{(i)} + \mathcal{O}(\theta^2) \end{aligned} \tag{54}$$

and

$$
\begin{aligned}
S_{\text{Matter,quarks}} &= \int \mathrm{d}^4x \\
&\times \left(\sum_i \left(\bar{Q}_{\mathrm{L}}^{(i)} + \bar{Q}_{\mathrm{L}}^{(i)1}\right) \star \mathrm{i}\left(\not{D}^{\mathrm{SM}} + \not{\Gamma}\right) \star \left(Q_{\mathrm{L}}^{(i)} + Q_{\mathrm{L}}^{(i)1}\right) \right. \\
&\left. + \sum_i \left(\bar{u}_{\mathrm{R}}^{(i)} + \bar{u}_{\mathrm{R}}^{(i)1}\right) \mathrm{i} \star \left(\not{D}^{\mathrm{SM}} + \not{\Gamma}\right) \star \left(u_{\mathrm{R}}^{(i)} + u_{\mathrm{R}}^{(i)1}\right) \right) \\
&+ \sum_i \left(\bar{d}_{\mathrm{R}}^{(i)} + \bar{d}_{\mathrm{R}}^{(i)1}\right) \mathrm{i} \star \left(\not{D}^{\mathrm{SM}} + \not{\Gamma}\right) \star \left(d_{\mathrm{R}}^{(i)} + d_{\mathrm{R}}^{(i)1}\right) \\
&+ \mathcal{O}(\theta^2) \\
&= \int \mathrm{d}^4x \sum_i \bar{Q}_{\mathrm{L}}^{(i)} \mathrm{i}\not{D}^{\mathrm{SM}} Q_{\mathrm{L}}^{(i)} - \frac{1}{4}\theta^{\mu\nu} \int \mathrm{d}^4x \\
&\times \sum_i \bar{Q}_{\mathrm{L}}^{(i)} (g' f_{\mu\nu} + g F_{\mu\nu}^L + g_S F_{\mu\nu}^S) \mathrm{i}\not{D}^{\mathrm{SM}} Q_{\mathrm{L}}^{(i)} - \frac{1}{2}\theta^{\mu\nu} \\
&\times \int \mathrm{d}^4x \sum_i \bar{Q}_{\mathrm{L}}^{(i)} \gamma^\alpha (g' f_{\alpha\mu} + g F_{\alpha\mu}^L + g_S F_{\alpha\mu}^S) \mathrm{i} D_\nu^{\mathrm{SM}} Q_{\mathrm{L}}^{(i)} \\
&+ \int \mathrm{d}^4x \sum_i \bar{u}_{\mathrm{R}}^{(i)} \mathrm{i}\not{D}^{\mathrm{SM}} u_{\mathrm{R}}^{(i)} \\
&- \frac{1}{4}\theta^{\mu\nu} \int \mathrm{d}^4x \sum_i \bar{u}_{\mathrm{R}}^{(i)} \left(g' f_{\mu\nu} + g_S F_{\mu\nu}^S\right) \mathrm{i}\not{D}^{\mathrm{SM}} u_{\mathrm{R}}^{(i)} \\
&- \frac{1}{2}\theta^{\mu\nu} \int \mathrm{d}^4x \sum_i \bar{u}_{\mathrm{R}}^{(i)} \gamma^\alpha \left(g' f_{\alpha\mu} + g_S F_{\mu\nu}^S\right) \mathrm{i} D_\nu^{\mathrm{SM}} u_{\mathrm{R}}^{(i)} \\
&+ \int \mathrm{d}^4x \sum_i \bar{d}_{\mathrm{R}}^{(i)} \mathrm{i}\not{D}^{\mathrm{SM}} d_{\mathrm{R}}^{(i)} \\
&- \frac{1}{4}\theta^{\mu\nu} \int \mathrm{d}^4x \sum_i \bar{d}_{\mathrm{R}}^{(i)} \left(g' f_{\mu\nu} + g_S F_{\mu\nu}^S\right) \mathrm{i}\not{D}^{\mathrm{SM}} d_{\mathrm{R}}^{(i)} \\
&- \frac{1}{2}\theta^{\mu\nu} \int \mathrm{d}^4x \sum_i \bar{d}_{\mathrm{R}}^{(i)} \gamma^\alpha \left(g' f_{\alpha\mu} + g_S F_{\mu\nu}^S\right) \mathrm{i} D_\nu^{\mathrm{SM}} d_{\mathrm{R}}^{(i)} \\
&+ \mathcal{O}(\theta^2). \qquad (55)
\end{aligned}
$$

We recover the commutative standard model, but some new interactions appear. The most striking feature are point-like interactions between gluons, electroweak bosons and quarks. For the gauge part of the action, one finds

$$
\begin{aligned}
S_{\text{gauge}} &= -\int \mathrm{d}^4x \frac{1}{2g'} \mathbf{tr_1} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} \\
&\quad - \int \mathrm{d}^4x \frac{1}{2g} \mathbf{tr_2} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} \\
&\quad - \int \mathrm{d}^4x \frac{1}{2g_S} \mathbf{tr_3} \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} \\
&= -\frac{1}{4} \int \mathrm{d}^4x f_{\mu\nu} f^{\mu\nu} - \frac{1}{2} \mathrm{Tr} \int \mathrm{d}^4x F_{\mu\nu}^L F^{L\mu\nu} \\
&\quad - g\theta^{\mu\nu} \mathrm{Tr} \int \mathrm{d}^4x F_{\mu\rho}^L F_{\nu\sigma}^L F^{L\rho\sigma} \\
&\quad - \frac{1}{2} \mathrm{Tr} \int \mathrm{d}^4x F_{\mu\nu}^S F^{S\mu\nu} \\
&\quad + \frac{1}{4} g_S \theta^{\mu\nu} \mathrm{Tr} \int \mathrm{d}^4x F_{\mu\nu}^S F_{\rho\sigma}^S F^{S\rho\sigma} \\
&\quad - g_S \theta^{\mu\nu} \mathrm{Tr} \int \mathrm{d}^4x F_{\mu\rho}^S F_{\nu\sigma}^S F^{S\rho\sigma} + \mathcal{O}(\theta^2). \qquad (56)
\end{aligned}
$$

The coefficients of the triple vertex in the $U(1)$ sector are also different from plain NCQED with a single electron. These coefficients depend on the representation we are choosing for the Y in the kinetic terms. For the simple choice that we have taken $\mathbf{tr_1} Y^3 = 0$ and this coefficient is zero. Note that a term

$$
+\frac{1}{4} g\theta^{\mu\nu} \mathrm{Tr} \int \mathrm{d}^4x F_{\mu\nu}^L F_{\rho\sigma}^L F^{L\rho\sigma} \qquad (57)
$$

vanishes, the trace over the three Pauli matrices yields $2\mathrm{i}\epsilon^{abc}$ and the sum $\epsilon^{abc} F_{\rho\sigma}^{bL} F^{cL\rho\sigma}$ vanishes. Note that because the trace over $\tau^3\tau^3\tau^3$ vanishes, there is also no cubic self-interaction term for the electromagnetic photon coming from the $SU(2)$ sector. Limits on non-commutative QED found from triple photon self-interactions do therefore not apply for the minimal non-commutative standard model.

As in the usual commutative standard model, the Higgs mechanism can be applied to break the $SU(2)_{\mathrm{L}} \times U(1)_Y$ gauge symmetry and thus to generate masses for the electroweak gauge bosons. The non-commutative action for a scalar field ϕ in the fundamental representation of $SU(2)_{\mathrm{L}}$ and with the hypercharge $Y = 1/2$ reads

$$
\begin{aligned}
S_{\text{Higgs}} &= \int \mathrm{d}^4x \left(\rho_0 \left(D_\mu \widehat{\Phi}\right)^\dagger \star \rho_0 \left(D^\mu \widehat{\Phi}\right) \right. \\
&\quad - \mu^2 \rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi}) \\
&\quad \left. - \lambda(\rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi})) \star (\rho_0(\widehat{\Phi})^\dagger \star \rho_0(\widehat{\Phi})) \right). \qquad (58)
\end{aligned}
$$

In the leading order of the expansion in θ, we obtain

$$
\begin{aligned}
S_{\text{Higgs}} &= \int \mathrm{d}^4x ((D_\mu^{\mathrm{SM}}\phi)^\dagger D^{\mathrm{SM}\mu}\phi - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)(\phi^\dagger\phi)) \\
&\quad + \int \mathrm{d}^4x \left((D_\mu^{\mathrm{SM}}\phi)^\dagger \right. \\
&\quad \times \left(D^{\mathrm{SM}\mu}\rho_0(\phi^1) + \frac{1}{2}\theta^{\alpha\beta}\partial_\alpha V^\mu \partial_\beta\phi + \Gamma^\mu\phi\right) \\
&\quad + \left(D_\mu^{\mathrm{SM}}\rho_0(\phi^1) + \frac{1}{2}\theta^{\alpha\beta}\partial_\alpha V_\mu \partial_\beta\phi + \Gamma_\mu\phi\right)^\dagger D^{SM\mu}\phi \\
&\quad + \frac{1}{4}\mu^2\theta^{\mu\nu}\phi^\dagger(g' f_{\mu\nu} + g F_{\mu\nu}^L)\phi \\
&\quad \left. - \lambda\mathrm{i}\theta^{\alpha\beta}\phi^\dagger\phi(D_\alpha^{\mathrm{SM}}\phi)^\dagger(D_\beta^{\mathrm{SM}}\phi) \right) + \mathcal{O}(\theta^2), \qquad (59)
\end{aligned}
$$

with

$$
\begin{aligned}
\Gamma_\mu = -\mathrm{i}V_\mu^1 = \mathrm{i}\frac{1}{4}\theta^{\alpha\beta}\Big\{ & g'\mathcal{A}_\alpha + gB_\alpha, g'\partial_\beta\mathcal{A}_\mu + g\partial_\beta B_\mu \\
& + g' f_{\beta\mu} + g F_{\beta\mu}^L \Big\} \qquad (60)
\end{aligned}
$$

and

$$\rho_0(\hat{\Phi}) = \phi + \rho_0(\phi^1) + \mathcal{O}(\theta^2), \quad (61)$$

where

$$\rho_0(\phi^1) = -\frac{1}{2}\theta^{\alpha\beta}(g'\mathcal{A}_\alpha + gB_\alpha)\partial_\beta\phi + \mathrm{i}\frac{1}{4}\theta^{\alpha\beta}(g'\mathcal{A}_\alpha + gB_\alpha)(g'\mathcal{A}_\beta + gB_\beta)\phi. \quad (62)$$

For $\mu^2 < 0$ the $SU(2)_\mathrm{L} \times U(1)_Y$ gauge symmetry is spontaneously broken to $U(1)_Q$, which is the gauge group describing the electromagnetic interactions. We have gauge freedom and take the so-called unitarity gauge

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \eta + v \end{pmatrix}, \quad (63)$$

where v is the vacuum expectation value. Since the leading order of the expansion of the non-commutative action corresponds to the standard model action, the Higgs mechanism generates masses for electroweak gauge bosons:

$$M_{W^\pm} = \frac{g}{2}v \quad \text{and} \quad M_Z = \frac{\sqrt{g^2 + g'^2}}{2}v, \quad (64)$$

where the physical mass eigenstates $W^\pm$, Z and A are as usual defined by

$$W^\pm_\mu = \frac{B^1_\mu \mp \mathrm{i}B^2_\mu}{\sqrt{2}},$$
$$Z_\mu = \frac{-g'\mathcal{A}_\mu + gB^3_\mu}{\sqrt{g^2 + g'^2}}$$

and

$$A_\mu = \frac{g\mathcal{A}_\mu + g'B^3_\mu}{\sqrt{g^2 + g'^2}}. \quad (65)$$

The Higgs mass is then given by $m^2_\eta = -2\mu^2$. Rewriting the term Γ_μ in terms of the mass eigenstates, using

$$B^3_\mu = \frac{gZ_\mu + g'A_\mu}{\sqrt{g^2 + g'^2}} \quad \text{and} \quad \mathcal{A}_\mu = \frac{gA_\mu - g'Z_\mu}{\sqrt{g^2 + g'^2}}, \quad (66)$$

one finds that besides the usual standard model couplings, numerous new couplings between the Higgs boson and the electroweak gauge bosons appear.

We note that the non-commutative version of the standard model is also compatible with the alternative to the Higgs mechanism proposed in [20].

The Yukawa couplings can then generate masses for the fermions; one has

$$\begin{aligned} S_\mathrm{Yukawa} = \int \mathrm{d}^4x \Bigg(& - \sum_{i,j=1}^{3} W^{ij} \Big((\bar{\widehat{L}}^{(i)}_\mathrm{L} \star \rho_\mathrm{L}(\widehat{\Phi})) \star \widehat{e}^{(j)}_\mathrm{R} \\ & + \bar{\widehat{e}}^{(i)}_\mathrm{R} \star (\rho_\mathrm{L}(\widehat{\Phi})^\dagger \star \widehat{L}^{(j)}_\mathrm{L}) \Big) \\ & - \sum_{i,j=1}^{3} G^{ij}_u \Big((\bar{\widehat{Q}}^{(i)}_\mathrm{L} \star \rho_{\bar{Q}}(\bar{\widehat{\Phi}})) \star \widehat{u}^{(j)}_\mathrm{R} \\ & + \bar{\widehat{u}}^{(i)}_\mathrm{R} \star (\rho_{\bar{Q}}(\bar{\widehat{\Phi}})^\dagger \star \widehat{Q}^{(j)}_\mathrm{L}) \Big) \\ & - \sum_{i,j=1}^{3} G^{ij}_d \Big((\bar{\widehat{Q}}^{(i)}_\mathrm{L} \star \rho_Q(\widehat{\Phi})) \star \widehat{d}^{(j)}_\mathrm{R} \\ & + \bar{\widehat{d}}^{(i)}_\mathrm{R} \star (\rho_Q(\widehat{\Phi})^\dagger \star \widehat{Q}^{(j)}_\mathrm{L}) \Big) \Bigg), \end{aligned} \quad (67)$$

with $\widehat{\Phi}[\Phi, V, V']$ as given in (34)–(36). The sum runs over the different generations. The leading order expansion is

$$\begin{aligned} S_\mathrm{Yukawa} = {} & S^\mathrm{SM}_\mathrm{Yukawa} \\ & - \int \mathrm{d}^4x \Bigg(\sum_{i,j=1}^{3} W^{ij} \Big((\bar{L}^i_\mathrm{L}\phi)e^{1j}_\mathrm{R} + (\bar{L}^i_\mathrm{L}\rho_\mathrm{L}(\phi^1))e^j_\mathrm{R} \\ & + (\bar{L}^{1i}_\mathrm{L}\phi)e^j_\mathrm{R} + \mathrm{i}\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha L^i_\mathrm{L}\partial_\beta\phi e^j_\mathrm{R} + \bar{e}^i_\mathrm{R}(\phi^\dagger L^{1j}_\mathrm{L}) \\ & + \bar{e}^i_\mathrm{R}(\rho_\mathrm{L}(\phi^1)^\dagger L^j_\mathrm{L}) + \bar{e}^{1i}_\mathrm{R}(\phi^\dagger L^j_\mathrm{L}) + \mathrm{i}\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha e^i_\mathrm{R}\partial_\beta\phi^\dagger L^j_\mathrm{L} \Big) \\ & - \sum_{i,j=1}^{3} G^{ij}_u \Big((\bar{Q}^i_\mathrm{L}\bar{\phi})u^{1j}_\mathrm{R} + (\bar{Q}^i_\mathrm{L}\rho_{\bar{Q}}(\bar{\phi}^1))u^j_\mathrm{R} + (\bar{Q}^{1i}_\mathrm{L}\bar{\phi})u^j_\mathrm{R} \\ & + \mathrm{i}\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha Q^i_\mathrm{L}\partial_\beta\bar{\phi}u^j_\mathrm{R} + \bar{u}^i_\mathrm{R}(\bar{\phi}^\dagger Q^{1j}_\mathrm{L}) + \bar{u}^i_\mathrm{R}(\rho_{\bar{Q}}(\bar{\phi}^1)^\dagger Q^j_\mathrm{L}) \\ & + \bar{u}^{1i}_\mathrm{R}(\bar{\phi}^\dagger Q^j_\mathrm{L}) + \mathrm{i}\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha u^i_\mathrm{R}\partial_\beta\bar{\phi}^\dagger Q^j_\mathrm{L} \Big) \\ & - \sum_{i,j=1}^{3} G^{ij}_d \Big((\bar{Q}^i_\mathrm{L}\phi)d^{1j}_\mathrm{R} + (\bar{Q}^i_\mathrm{L}\rho_Q(\phi^1))d^j_\mathrm{R} + (\bar{Q}^{1i}_\mathrm{L}\phi)d^j_\mathrm{R} \\ & + \mathrm{i}\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha Q^i_\mathrm{L}\partial_\beta\phi d^j_\mathrm{R} + \bar{d}^i_\mathrm{R}(\phi^\dagger Q^{1j}_\mathrm{L}) + \bar{d}^i_\mathrm{R}(\rho_Q(\phi^1)^\dagger Q^j_\mathrm{L}) \\ & + \bar{d}^{1i}_\mathrm{R}(\phi^\dagger Q^j_\mathrm{L}) + \mathrm{i}\frac{1}{2}\theta^{\alpha\beta}\partial_\alpha \bar{d}^i_\mathrm{R}\partial_\beta\phi^\dagger Q^j_\mathrm{L} \Big) \Bigg) + \mathcal{O}(\theta^2), \end{aligned} \quad (68)$$

where L^i_L stands for a left-handed leptonic doublet of the ith generation, e^i_R for a leptonic singlet of the ith generation, Q^i_L for a left-handed quark doublet of the ith generation, u^i_R for a right-handed up-type quark singlet of the ith and d^i_R stands for a right-handed down-type quark singlet of the ith generation. We used

$$\rho(\Phi) = \phi + \rho(\phi^1) + \mathcal{O}(\theta^2), \quad (69)$$

where ρ stands for ρ_L, ρ_Q and $\rho_{\bar{Q}}$, respectively. $\rho(\phi^1)$ is given by (21),

$$\begin{aligned} \rho(\phi^1) = {} & \frac{1}{2}\theta^{\mu\nu}\rho(V_\nu)\left(\partial_\mu\phi - \frac{\mathrm{i}}{2}\rho(V_\mu)\phi + \frac{\mathrm{i}}{2}\phi\rho(V'_\mu)\right) \\ & + \frac{1}{2}\theta^{\mu\nu}\left(\partial_\mu\phi - \frac{\mathrm{i}}{2}\rho(V_\mu)\phi + \frac{\mathrm{i}}{2}\phi\rho(V'_\mu)\right)\rho(V'_\nu). \end{aligned} \quad (70)$$

Once again we recover the standard model, but some new interactions arise. The Yukawa coupling matrices can be diagonalized using biunitary transformations. We thus obtain a Cabibbo–Kobayashi–Maskawa matrix in the charged currents, as in the standard model and as long as right-handed neutrinos are absent, we do not predict lepton flavor changing currents. We give the Lagrangian for the charged currents in Appendix A and that for the neutral currents in Appendix B. Clearly, flavor physics is much richer than in the standard model on a commutative space.

5 Non-commutative quantum chromodynamics

The method developed in [4] has been applied to non-commutative quantum chromodynamics NCQCD already [21]. But the authors of [21] have only considered the gauge group $SU(3)_C$ instead of $SU(3)_C \times SU(2)_\mathrm{L} \times U(1)_Y$ which is the relevant gauge group to describe charged quarks. Our results are thus different since the quarks are not only in the fundamental representation of $SU(3)$ but they are also charged under $SU(2)_\mathrm{L} \times U(1)_Y$. This implies in particular that parity is broken in NCQCD in the leading order of the expansion in θ as the left-handed quarks are charged under $SU(2)_\mathrm{L}$. One thus has to treat the right-handed and left-handed quarks separately. The expansion for the non-commutative quarks is thus of the form (42) for a left-handed quark Q_L and of the form (44) for a right-handed quark Q_R. The non-commutative action has actually already been given previously in (54), although it appears in a hidden fashion.

6 Discussion of the model

We have shown in Sect. 4 that the commutative electroweak standard model comes out as the zeroth order of the expansion in $\theta^{\mu\nu}$ of the action of the non-commutative standard model (NCSM). Although we have considered a minimal non-commutative standard model, there is a basic difference between the commutative and non-commutative versions: in the non-commutative model, the different interactions cannot be considered separately as the master field V_μ, which is a superposition of the different gauge fields, has to be introduced. In the leading order of the expansion in θ, we find that the gauge bosons of the different gauge groups decouple. But because the quarks are charged under $SU(3)_C$ as well as under $SU(2)_\mathrm{L} \times U(1)_Y$ some new vertices appear where the gauge bosons of different gauge groups are connected to the quarks. In the minimal non-commutative standard model, a kind of mixing or unification between all the interactions appears as we have vertices where e.g. $SU(3)_C$ gauge bosons couple to the $U(1)_Y$ gauge boson and to quarks. This type of unification implies that parity is broken in NCQCD.

Up to the order considered we do not find couplings of neutral particles like the Higgs boson to the electromagnetic photon in the minimal version of the NCSM. We also find new vertices in the pure gauge sector. In contradiction with naive expectations the $U(1)_Y$ gauge boson does not have a self-interacting vertex to the order considered, but one finds vertices with five and six gauge bosons for the gauge group $SU(3)_C$ and $SU(2)_\mathrm{L}$.

All the important features of the ordinary standard model can be implemented in the model, in particular the Higgs mechanism and the Yukawa sector. Biunitary transformations can be applied to diagonalize the matrices of Yukawa couplings.

Recently a model based on the gauge group $U(3) \times U(2) \times U(1)$ was proposed [22]. This model involves a clever extra Higgs mechanism to deal with the problems of charge quantization and tensor products, but it contains two gauge bosons which are not present in the usual standard model. What we are doing is fundamentally different, as we are considering the standard model gauge group $SU(3) \times SU(2) \times U(1)$ directly. We thus have proposed a *minimal* non-commutative extension of the standard model.

We have presented the first order expansion in $\theta^{\mu\nu}$ of the non-commutative standard model, which only represents a low energy effective theory. The limits that can be found in the literature on the combination $\Lambda\theta$ are based on the assumption that $\theta^{\mu\nu}$ is constant [23]; clearly the limits are much weaker if the assumption is relaxed. As in the case of chiral perturbation theory, the effects are expected to be small for light particles. But they could be sizable for heavy particles. In particular it is conceivable that a phase transition occurs a high energy; Nature could be non-commutative above that scale but commutative under the scale of this phase transition.

Clearly the standard model on a non-commutative space-time predicts a lot of new physics beyond the standard model. In particular as we have seen, we expect the charged and neutral currents to be considerably affected by non-commutative physics. The extraction of the CKM matrix elements and in particular of the phase at the origin of CP-violation would be strongly influenced by that type of new physics. One expects that the effects should become larger with the mass of the decaying particle, especially if a phase transition exists. This might explain why the standard model on a commutative space can accommodate accurately CP-violation in the kaon system although large non-commutative effects could show up in e.g. the B-meson system.

High energy cosmic rays are also a place to probe non-commutative physics. It has been proposed by Coleman and Glashow [24] that a violation of Lorentz invariance could explain this phenomenon.

There are indications that our model may be renormalizable to all orders in the coupling constants and in θ: A study in the framework of non-commutative quantum electrodynamics [8] has shown that the photon self-energy is renormalizable to all order. But a proof of the renormalizability of our model is still to be furnished.

The problem of ultra-violet and infra-red mixing, which plagues non-commutative quantum field theories [25], should be reconsidered in the framework of the Seiberg–Witten expansion used in our approach [26]. Note that the ultra-violet and infra-red mixing is absent in the case of Φ^4 theory on a fuzzy sphere [27], where the quantization has been performed via path integrals.

7 Conclusions

We have considered the minimal non-commutative extension of the standard model (NCSM) and have calculated the first order expansion of the model in $\theta^{\mu\nu}$. This required one to solve two problems: the $U(1)$ charge quantization and the application of the Seiberg–Witten method to a tensor product of groups. The trace over the field strength has to be defined properly. We obtain a low energy effective theory valid for small transferred momentum, in that sense it is the analog of chiral perturbation theory for quantum chromodynamics. The zeroth order expansion is the commutative standard model. This model has the same number of free coupling constants and fields as the usual standard model.

We find that the most striking feature of the model is a new type of unification as all interactions have to be considered simultaneously. We have found that the Higgs boson does not couple to the electroweak photon in the minimal NCSM and that new effects in the charged and neutral currents are expected. This will affect the extraction of the CKM matrix parameters and in particular of the CP-violating phase. Neutral decays of heavy particle, e.g. of the b and t quarks might also reveal the non-commutative nature of space-time. New vertices appear in QCD. We find a point-like interaction between two quarks, a gluon and a photon, thus opening new decay modes for hadrons. Parity is violated in the leading order of the expansion in the non-commutative parameter θ.

The non-commutative standard model represents a very natural extension of the standard model; it could improve some of its problems, like naturalness and the so-called hierarchy problem and it represents a natural attempt to include the effects of quantum gravity in particle physics.

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Appendix

A Charged currents

In this section we give the explicit formulas for the electroweak charged currents in the leading order of the expansion in θ.

$$\mathcal{L} = (\,\bar{u} \quad \bar{c} \quad \bar{t}\,)_{\mathrm{L}}\, V_{\mathrm{CKM}} J_1 \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\mathrm{L}} + (\,\bar{d} \quad \bar{s} \quad \bar{b}\,)_{\mathrm{L}}\, V_{\mathrm{CKM}}^{\dagger} J_2 \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{\mathrm{L}}, \tag{71}$$

with

$$\begin{aligned} J_1 = {} & \frac{1}{\sqrt{2}} g \not{W}^{+} + \left(\frac{1}{2}\theta^{\mu\nu}\gamma^{\alpha} + \theta^{\nu\alpha}\gamma^{\mu}\right) \\ & \times\Bigg(\Big(-\frac{\sqrt{2}}{4} Y g' g (\cos\theta_{\mathrm{W}}\partial_\mu A_\nu - \cos\theta_{\mathrm{W}}\partial_\nu A_\mu \\ & - \sin\theta_{\mathrm{W}}\partial_\mu Z_\nu + \sin\theta_{\mathrm{W}}\partial_\nu Z_\mu) W_\alpha^{+}\Big) \\ & + g\frac{\sqrt{2}}{8}\Big(\partial_\mu W_\nu^{+} - \partial_\nu W_\mu^{+} \\ & - 2\mathrm{i} g\Big(\cos\theta_{\mathrm{W}} Z_\mu W_\nu^{+} + \sin\theta_{\mathrm{W}} A_\mu W_\nu^{+} \\ & - \cos\theta_{\mathrm{W}} W_\mu^{+} Z_\nu - \sin\theta_{\mathrm{W}} W_\mu^{+} A_\nu\Big)\Big) \\ & \times\Big(-2\mathrm{i}\partial_\alpha + 2Y g' \sin\theta_{\mathrm{W}} Z_\alpha \\ & - 2Y g' \cos\theta_{\mathrm{W}} A_\alpha + g\cos\theta_{\mathrm{W}} Z_\alpha + g\sin\theta_{\mathrm{W}} A_\alpha\Big) \\ & - \frac{\sqrt{2}}{8} g^2 \Big(\cos\theta_{\mathrm{W}}\partial_\mu Z_\nu - \cos\theta_{\mathrm{W}}\partial_\nu Z_\mu \\ & + \sin\theta_{\mathrm{W}}\partial_\mu A_\nu - \sin\theta_{\mathrm{W}}\partial_\nu A_\mu \\ & - 2\mathrm{i} g (W_\mu^{+} W_\nu^{-} - W_\nu^{+} W_\mu^{-})\Big) W_\alpha^{+}\Bigg) \end{aligned} \tag{72}$$

and

$$\begin{aligned} J_2 = {} & \frac{1}{\sqrt{2}} g \not{W}^{-} + \left(\frac{1}{2}\theta^{\mu\nu}\gamma^{\alpha} + \theta^{\nu\alpha}\gamma^{\mu}\right) \\ & \times\Bigg(\Big(-\frac{\sqrt{2}}{4} Y g' g (\cos\theta_{\mathrm{W}}\partial_\mu A_\nu - \cos\theta_{\mathrm{W}}\partial_\nu A_\mu \\ & - \sin\theta_{\mathrm{W}}\partial_\mu Z_\nu + \sin\theta_{\mathrm{W}}\partial_\nu Z_\mu) W_\alpha^{-}\Big) \\ & + g\frac{\sqrt{2}}{8}\Big(\partial_\mu W_\nu^{-} - \partial_\nu W_\mu^{-} \\ & - 2\mathrm{i} g\Big(\cos\theta_{\mathrm{W}} W_\mu^{-} Z_\nu + \sin\theta_{\mathrm{W}} W_\mu^{-} A_\nu \\ & - \cos\theta_{\mathrm{W}} Z_\mu W_\nu^{-} - \sin\theta_{\mathrm{W}} A_\mu W_\nu^{+}\Big)\Big) \\ & \times\Big(-2\mathrm{i}\partial_\alpha + 2Y g' \sin\theta_{\mathrm{W}} Z_\alpha - 2Y g' \cos\theta_{\mathrm{W}} A_\alpha \\ & - g\cos\theta_{\mathrm{W}} Z_\alpha - g\sin\theta_{\mathrm{W}} A_\alpha\Big) \\ & - \frac{\sqrt{2}}{8} g^2 \Big(\cos\theta_{\mathrm{W}}\partial_\mu Z_\nu - \cos\theta_{\mathrm{W}}\partial_\nu Z_\mu \\ & + \sin\theta_{\mathrm{W}}\partial_\mu A_\nu - \sin\theta_{\mathrm{W}}\partial_\nu A_\mu \\ & - 2\mathrm{i} g (W_\mu^{+} W_\nu^{-} - W_\nu^{+} W_\mu^{-})\Big) W_\alpha^{-}\Bigg). \end{aligned} \tag{73}$$

Note that we have not included the interactions with the gluons in the "electroweak" charged currents.

B Neutral currents

In this appendix we give the explicit formula for the neutral current in the leading order of the expansion in θ:

$$\mathcal{L}_{\rm nc} = \mathcal{L}_{\rm nc}^{\rm SM} - {\rm i}\frac{1}{2}\sum_i \bar{u}_{\rm L}^{(i)}\left(\frac{1}{2}\theta^{\mu\nu}\gamma^\alpha + \theta^{\nu\alpha}\gamma^\mu\right)$$
$$\times\Bigg(\Big(\cos\theta_{\rm W}\partial_\mu A_\nu - \cos\theta_{\rm W}\partial_\nu A_\mu$$
$$-\sin\theta_{\rm W}\partial_\mu Z_\nu + \sin\theta_{\rm W}\partial_\nu Z_\mu\Big)$$
$$\times\Big(g'Y\partial_\alpha - {\rm i}Y^2 g'^2\cos\theta_{\rm W}A_\alpha + {\rm i}Y^2 g'^2 \sin\theta_{\rm W}Z_\alpha$$
$$-{\rm i}\frac{1}{2}Yg'g\cos\theta_{\rm W}Z_\alpha - {\rm i}\frac{1}{2}Yg'g\sin\theta_{\rm W}A_\alpha\Big)$$
$$+\frac{1}{2}\Big(\cos\theta_{\rm W}\partial_\mu Z_\nu - \cos\theta_{\rm W}\partial_\nu Z_\mu$$
$$+\sin\theta_{\rm W}\partial_\mu A_\nu - \sin\theta_{\rm W}\partial_\nu A_\mu$$
$$-2{\rm i}g(W_\mu^+W_\nu^- - W_\nu^+W_\mu^-)\Big)\Big(g\partial_\alpha - iYg'g\cos\theta_{\rm W}A_\alpha$$
$$+{\rm i}Yg'g\cos\theta_{\rm W}Z_\alpha - \frac{1}{2}{\rm i}g^2\cos\theta_{\rm W}Z_\alpha - \frac{1}{2}{\rm i}g^2\sin\theta_{\rm W}A_\alpha\Big)$$
$$-\frac{\rm i}{2}g^2\Big(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+$$
$$-2{\rm i}g\Big(\cos\theta_{\rm W}Z_\mu W_\nu^+ + \sin\theta_{\rm W}A_\mu W_\nu^+ - W_\mu^+\cos\theta_{\rm W}Z_\nu$$
$$-W_\mu^+\sin\theta_{\rm W}A_\nu\Big)\Big)W_\alpha^-\Bigg)u_{\rm L}^{(i)}$$
$$-{\rm i}\frac{1}{2}\sum_i \bar{u}_{\rm R}^{(i)}\left(\frac{1}{2}\theta^{\mu\nu}\gamma^\alpha + \theta^{\nu\alpha}\gamma^\mu\right)$$
$$\times\Big(\Big(\cos\theta_{\rm W}\partial_\mu A_\nu - \cos\theta_{\rm W}\partial_\nu A_\mu$$
$$-\sin\theta_{\rm W}\partial_\mu Z_\nu + \sin\theta_{\rm W}\partial_\nu Z_\mu\Big)$$
$$\times\Big(g'Y\partial_\alpha - {\rm i}Y^2g'^2\cos\theta_{\rm W}A_\alpha + {\rm i}Y^2g'^2\sin\theta_{\rm W}Z_\alpha\Big)\Big)u_{\rm R}^{(i)}$$
$$-{\rm i}\frac{1}{2}\sum_i \bar{d}_{\rm L}^{(i)}\left(\frac{1}{2}\theta^{\mu\nu}\gamma^\alpha + \theta^{\nu\alpha}\gamma^\mu\right)$$
$$\times\Bigg(\Big(\cos\theta_{\rm W}\partial_\mu A_\nu - \cos\theta_{\rm W}\partial_\nu A_\mu$$
$$-\sin\theta_{\rm W}\partial_\mu Z_\nu + \sin\theta_{\rm W}\partial_\nu Z_\mu\Big)$$
$$\times\Big(g'Y\partial_\alpha - {\rm i}Y^2g'^2\cos\theta_{\rm W}A_\alpha + {\rm i}Y^2g'^2\sin\theta_{\rm W}Z_\alpha$$
$$-{\rm i}\frac{1}{2}Yg'g\cos\theta_{\rm W}Z_\alpha - {\rm i}\frac{1}{2}Yg'g\sin\theta_{\rm W}A_\alpha\Big)$$
$$-\frac{1}{2}\Big(\cos\theta_{\rm W}\partial_\mu Z_\nu - \cos\theta_{\rm W}\partial_\nu Z_\mu$$
$$+\sin\theta_{\rm W}\partial_\mu A_\nu - \sin\theta_{\rm W}\partial_\nu A_\mu$$
$$-2{\rm i}g(W_\mu^+W_\nu^- - W_\nu^+W_\mu^-)\Big)\Big(g\partial_\alpha - {\rm i}Yg'g\cos\theta_{\rm W}A_\alpha$$
$$+{\rm i}Yg'g\cos\theta_{\rm W}Z_\alpha + \frac{1}{2}{\rm i}g^2\cos\theta_{\rm W}Z_\alpha + \frac{1}{2}{\rm i}g^2\sin\theta_{\rm W}A_\alpha\Big)$$
$$-\frac{\rm i}{2}g^2\Big(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-$$
$$+2{\rm i}g\Big(\cos\theta_{\rm W}Z_\mu W_\nu^- + \sin\theta_{\rm W}A_\mu W_\nu^-$$
$$-W_\mu^-\cos\theta_{\rm W}Z_\nu - W_\mu^-\sin\theta_{\rm W}A_\nu\Big)\Big)W_\alpha^+\Bigg)d_{\rm L}^{(i)}$$
$$-{\rm i}\frac{1}{2}\sum_i \bar{d}_{\rm R}^{(i)}\left(\frac{1}{2}\theta^{\mu\nu}\gamma^\alpha + \theta^{\nu\alpha}\gamma^\mu\right)$$
$$\times\Bigg(\Big(\cos\theta_{\rm W}\partial_\mu A_\nu - \cos\theta_{\rm W}\partial_\nu A_\mu - \sin\theta_{\rm W}\partial_\mu Z_\nu$$
$$+\sin\theta_{\rm W}\partial_\nu Z_\mu\Big)\Big(g'Y\partial_\alpha - {\rm i}Y^2g'^2\cos\theta_{\rm W}A_\alpha$$
$$+{\rm i}Y^2g'^2\sin\theta_{\rm W}Z_\alpha\Big)\Bigg)d_{\rm R}^{(i)}. \quad (74)$$

Note that we have not included the interactions with the gluons in the "electroweak" neutral currents.

C Kinetic terms for the gauge bosons

Here we will discuss the kinetic terms for the gauge bosons in more detail and will propose an alternative to the choice presented in the main part of this paper.

Let us reconsider the discussion of charge in non-commutative QED in Sect. 2.3. We found that without knowledge of the existence of Seiberg–Witten maps we would conclude that we need to introduce a separate physical gauge field $\hat{a}_\mu^{(n)}$ for every charge $q^{(n)}$ in the model. Equivalently we can also say that the mathematical field $\widehat{A}_\mu$ depends nonlinearly on the charge operator Q, i.e., it is enveloping algebra-valued. Then $\widehat{A}_\mu\widehat{\Psi}^{(n)} \equiv eq^{(n)}\hat{a}_\mu^{(n)}\widehat{\Psi}^{(n)}$ with $\hat{a}_\mu^{(n)} \neq \hat{a}_\mu^{(m)}$ for $q^{(n)} \neq q^{(m)}$. The gauge field $\hat{a}_\mu^{(n)}$ appears in the covariant derivative

$$\widehat{D}_\mu\widehat{\Psi}^{(n)} = \partial_\mu\widehat{\Psi}^{(n)} - {\rm i}eq^{(n)}\hat{a}_\mu^{(n)} \star \widehat{\Psi}^{(n)}. \quad (75)$$

It is natural to provide a kinetic term for each of these gauge fields $\hat{a}_\mu^{(n)}$, i.e.,

$$S_{\rm NCQED} = -\frac{1}{4N}\int {\rm d}^4x \sum_{n=1}^{N} \hat{f}_{\mu\nu}^{(n)} \star \hat{f}^{(n)\mu\nu} \quad (+\text{fermions}), \quad (76)$$

where the field strength $\hat{f}_{\mu\nu}^{(n)}$ corresponding to the gauge field $\hat{a}_\mu^{(n)}$ is determined by

$$\widehat{F}_{\mu\nu}\widehat{\Psi}^{(n)} \equiv eq^{(n)}\hat{f}_{\mu\nu}^{(n)}\widehat{\Psi}^{(n)}, \quad (77)$$

with $\widehat{F}_{\mu\nu} = \partial_\mu\widehat{A}_\nu - \partial_\nu\widehat{A}_\mu - {\rm i}[\widehat{A}_\mu \stackrel{\star}{,} \widehat{A}_\nu]$. The factor $1/N$ in front of the action takes care of the correct commutative limit. We can also write the action in terms of $\widehat{F}_{\mu\nu}$, the charge operator Q and an appropriately normalized trace **Tr** over the states $\widehat{\Psi}^{(n)}$:

$$S_{\text{NCQED}} = -\frac{1}{2}\int \mathrm{d}^4x \mathbf{Tr}\frac{1}{(eQ)^2}\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu} \quad (\text{+fermions}). \tag{78}$$

From a physical point of view there is no reason to use the same coupling constant e for all gauge fields $\hat{a}_\mu^{(n)}$ in (75). We could as well introduce individual coupling constants and correspondingly rescaled fields $\hat{a}_\mu^{\prime(n)}$, $\hat{f}_{\mu\nu}^{\prime(n)}$. This leads to an alternative action:

$$S'_{\text{NCQED}} = -\frac{1}{2}\int \mathrm{d}^4x \mathbf{Tr}\frac{1}{G^2}\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu} \quad (\text{+fermions}), \tag{79}$$

where G is an operator that is a function of the charge operator Q and certain constants g_n, such that

$$G\widehat{\Psi}^{(n)} \propto g_n\widehat{\Psi}^{(n)}$$

and

$$\mathbf{Tr}\frac{1}{G^2}\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu} = \frac{1}{N}\sum_{n=1}^{N}\frac{e^2}{g_n^2}(q^{(n)})^2\hat{f}_{\mu\nu}^{\prime(n)}\star\hat{f}^{\prime(n)\mu\nu}. \tag{80}$$

The usual coupling constant e can be expressed in terms of the g_n by

$$\mathbf{Tr}\frac{1}{G^2}Q^2 = \sum_{n=1}^{N}\frac{1}{g_n^2}(q^{(n)})^2 = \frac{1}{2e^2}. \tag{81}$$

In the classical limit only this combination of the g_n is relevant.

We have chosen a set-up that can be directly generalized to more general gauge theories including the standard model. The action for non-Abelian non-commutative gauge bosons is

$$S_{\text{gauge}} = -\frac{1}{2}\int \mathrm{d}^4x \mathbf{Tr}\frac{1}{\mathbf{G}^2}\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu}, \tag{82}$$

with the non-commutative field strength $\widehat{F}_{\mu\nu}$, an appropriate trace **Tr** and an operator **G**. This operator must commute with all generators (Y, T_{L}^a, T_S^b) of the gauge group so that it does not spoil the trace property of **Tr**. From what we have discussed above, it is natural to choose a trace over all the particles (with different quantum numbers) in the model that have covariant derivatives acting on them. In the standard model these are for each generation five multiplets of fermions and one Higgs multiplet; see Table 1. The operator **G** is in general a function of Y and the Casimirs of $SU(2)$ and $SU(3)$. However, due to the special asignment of hypercharges in the standard model it is possible to express **G** just in terms of Y and six constants $g_1, \ldots, g_6$ corresponding to the six multiplets. In the classical limit only certain combinations of these six constants, corresponding to the usual coupling constants g', g, g_S, are relevant. The relation is given by the following equations:

$$\begin{gathered}\frac{1}{g_1^2}+\frac{1}{2g_2^2}+\frac{4}{3g_3^2}+\frac{1}{3g_4^2}+\frac{1}{6g_5^2}+\frac{1}{2g_6^2}=\frac{1}{2g'^2},\\ \frac{1}{g_2^2}+\frac{3}{g_5^2}+\frac{1}{g_6^2}=\frac{1}{g^2}, \quad \frac{1}{g_3^2}+\frac{1}{g_4^2}+\frac{2}{g_5^2}=\frac{1}{g_S^2}.\end{gathered} \tag{83}$$

These three equations define for fixed g', g, g_S a three-dimensional simplex in the six-dimensional moduli space spanned by $1/g_1^2, \ldots, 1/g_6^2$. The remaining three degrees of freedom become relevant at order θ in the expansion of the non-commutative action. Interesting are in particular the following traces corresponding to triple gauge boson vertices:

$$\begin{aligned}\mathbf{Tr}\frac{1}{\mathbf{G}^2}Y^3 &= -\frac{1}{g_1^2}-\frac{1}{4g_2^2}+\frac{8}{9g_3^2}-\frac{1}{9g_4^2}+\frac{1}{36g_5^2}+\frac{1}{4g_6^2},\\ \mathbf{Tr}\frac{1}{\mathbf{G}^2}YT_{\mathrm{L}}^aT_{\mathrm{L}}^b &= \frac{1}{2}\delta^{ab}\left(-\frac{1}{2g_2^2}+\frac{1}{2g_5^2}+\frac{1}{2g_6^2}\right),\\ \mathbf{Tr}\frac{1}{\mathbf{G}^2}YT_S^cT_S^d &= \frac{1}{2}\delta^{cd}\left(\frac{2}{3g_3^2}-\frac{1}{3g_4^2}+\frac{1}{3g_5^2}\right).\end{aligned} \tag{84}$$

We could choose, e.g., to maximize the traces over Y^3 and $YT_{\mathrm{L}}^aT_{\mathrm{L}}^b$. This gives $1/g_1^2 = 1/(2g'^2) - 4/(3g_S^2) - 1/(2g^2)$, $1/g_3^2 = 1/g_S^2$, $1/g_6^2 = 1/g^2$, $1/g_2^2 = 1/g_4^2 = 1/g_5^2 = 0$ and

$$\begin{aligned}\mathbf{Tr}\frac{1}{\mathbf{G}^2}Y^3 &= -\frac{1}{2g'^2}+\frac{3}{4g^2}+\frac{20}{9g_S^2},\\ \mathbf{Tr}\frac{1}{\mathbf{G}^2}YT_{\mathrm{L}}^aT_{\mathrm{L}}^b &= \frac{1}{4g^2}\delta^{ab},\\ \mathbf{Tr}\frac{1}{\mathbf{G}^2}YT_S^cT_S^d &= \frac{2}{6g_S^2}\delta^{cd}.\end{aligned}$$

In the scheme that we have presented in the main part of this paper all three traces are zero. One consequence is that while non-commutativity does not *require* a triple photon vertex, such a vertex is nevertheless consistent with non-commutativity. It is important to note that the values of all three traces are bounded for any choice of constants.

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